Analysis of students' mental construction in understanding the concept of partial derivatives based on action-process-object-schema theory

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ABSTRACT

Purpose: This study aims to examine the errors that students made when using the APOS (Action-Process-Object-Schema) theory to create their knowledge of partial derivatives.

Design/Methodology/Approach: The method used was descriptive-qualitative including data collection through tests, interviews and documentation. Triangulation techniques were used in data validity procedures to ensure the reliability of the data.

Findings: The results showed that high, medium and low ability students had an understanding of the action, process and object stages in determining the partial derivatives. However, medium-ability students experienced a deficiency in understanding the object stage while low-ability students showed shortcomings in understanding the process and object stages. Several mistakes were made in determining partial derivatives of algebraic and trigonometric functions including incorrectly selecting and using the procedures of addition, subtraction, multiplication, division and chain rules as well as writing conclusions.

Conclusion: The dominant types of errors in determining partial derivatives were executive (ExE) and process skill (PE).

Recommendations: Future studies were expected to modify the genetic decomposition of partial derivatives and expand the classification of errors in data analysis.

Keywords: *APOS theory, Errors, Initial genetic decomposition, Partial derivative.*

1. INTRODUCTION

Calculus has basic concepts considered prerequisites for all university courses (Bressoud, 2021; Nongharnpituk, Yonwilad, & Khansila, 2022). Students attain optimal calculus learning results to avoid lecture failure and to remain competitive in the workforce (Ayebo, Ukkelberg, & Assuah, 2017). Nonetheless, most students struggle to grasp the ideas of calculus (Hurdle & Mogilski, 2022). They do not understand materials such as functions (Syarifuddin & Sari, 2021), limits (Bansilal & Mkhwanazi, 2022), continuity (Perfekt, 2021), derivatives (Bangaru et al., 2021; Toh, Tay, & Tong, 2021), integrals (Fernandez & Mohammed, 2021) and rate of change (Avgerinos & Remoundou, 2021; Frank & Thompson, 2021). Students do not have a clear picture of functions such as domain and range (McDowell, 2021). Olivier (1989) stated that misconceptions about learning were closely related to mental structure. Therefore, experts in mathematics education are trying to overcome the misconception by using the APOS theory (Borji, Alamolhodaei, & Radmehr, 2018; Burns-Childers & Vidakovic, 2018; Maharaj, 2013; Siyepu, 2015).

One of the calculus subjects that has numerous implications in a variety of fields is derivative (Moru, 2020). A considerable number of students encounter challenges in comprehending the topic despite its widespread utility (Orton, 1983; Uygur & Özdaş, 2005). Previous studies reported several related difficulties including determining derivatives with function composition (Tall, 1993) and chain rules (Gordon, 2005; Uygur & Özdaş, 2005). Maharaj (2013) found that most students answered $f'(x) = \frac{1}{3x^2+1}$ when determining the derivative of $f(x) = \ln(3x^2 + 1)$. This is due to students' inability to understand the relationships between different mathematical disciplines.

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(Rismaini & Devita, 2023). Therefore, the basic concept of the derivative is not comprehended (Borji et al., 2018; Dominguez, Barniol, & Zavala, 2017).

In the field of calculus, another critical concept is the partial derivative. This term pertains to situations where two or more independent variables are concurrently considered distinct from the ordinary derivative. For example, $z = f(x, y)$ is a function with two independent variables x and y. Since x and y are independent variables, there are several possibilities, namely (1) y is considered fixed while x is changing, (2) x is considered fixed while y is changing and (3) x and y change together. In determining partial derivatives, students always use the rules for the derivative of a one-variable function. For example, when determining $\frac{\partial z}{\partial x}$ from $z = f(x, y)$, the process includes treating y as a constant while specifically varying the variable x. The method comprises treating x as a constant while selectively varying y to obtain $\frac{\partial z}{\partial y}$ from $z = f(x, y)$.

Calculus material is difficult for students to understand because of the rigorous lecture style created by the use of modules and assignments which also hinders students' ability to develop their thought processes. This might be due to the module's presentation of common problems which does not promote students' thinking skills (Baye, Ayele, & Wondimuneh, 2021). In addition to imparting knowledge, a teacher's role also includes developing students' higher-order thinking skills (Purnomo, Sukestiyarno, Junaedi, & Agoestanto, 2024). The academic success of students is also impacted by this skill. Partial derivatives become very difficult when the derivative rules are not understood and this leads to numerous mistakes. These errors are caused by the failure to comprehend the procedures of addition, subtraction, multiplication, division and chain rules as well as a lack of accuracy in performing algebraic arithmetic operations. Consequently, it is essential to comprehend how students create their minds. The categorization of errors can assist lecturers and teachers in concentrating on creating educational strategies for handling obstacles to learning (Siyepu, 2015; Swan, 2001). As a result, teachers need to be able to create mathematical lessons that are both engaging and difficult for students (Fauzan, Harisman, & Sya'bani, 2022).

One of the theories used to analyze a student's concept construction process is the APOS (Action-Process-Object-Schema) theory. The philosophical foundation of APOS theory is social constructivism (Altieri & Schirmer, 2019; Prasetyo, Sukestiyarno, & Cahyono, 2021). Understanding how students learn numerous mathematical concepts is made easier by the use of this approach. Moreover, the idea describes comprehension level by creating and employing mental structures such as actions, processes, objects and schemas (Arnon et al., 2014). When understanding mathematical concepts, someone demands the appropriate mental architecture (Maharaj, 2013). Actions can be defined as transformations of objects, including explicit, step-by-step instructions for executing specific operations. Processes are internally occurring mental constructs acquired when an individual acts repeatedly. During the process level, people do not need an increased level of external stimulus. Objects are constructed when the process is complete and transformations are made. The collection of actions, processes and objects linked by assured shared principles constitutes a schema for a specific mathematical idea (García-Martínez & Parraguez, 2017; Syamsuri & Santosa, 2021).

Mathematical conceptions are developed by an organized set of mental actions called genetic decomposition (Zwanch, 2019). A specific mathematical principle should be learned by the role as a hypothetical model of mental constructions (Arnon et al., 2014). Genetic decomposition is a form of analysis when individuals describe mathematical problems according to the APOS theory framework. The analysis's findings demonstrate people's comprehension of those who make an effort to grasp mathematical ideas. Materials prepared based on genetic decomposition were proven to improve students' abstraction abilities in several courses.

APOS theory was applied in various prior studies to investigate mathematical understanding of functions (Bansilal, Brijlall, & Trigueros, 2017; Martínez-Planell & Trigueros, 2019), algebra (Harel, 2017), gradients (Nagle, Martínez-Planell, & Moore-Russo, 2019), limits (Baye et al., 2021), integrals (Borji & Martínez-Planell, 2023; Martínez-Planell & Trigueros, 2020), matrix (Figueroa, Possani, & Trigueros, 2018) and induction principles (García-Martínez & Parraguez, 2017). Nevertheless, there needs to be more studies using APOS theory to assess how well students understand the partial derivatives of the functions, making trigonometry difficult (Siyepu, 2015). There is also a limitation in relating the concept to errors made during construction, specifically Orton (1983) and Newman (1977) errors.

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For this reason, student answers should be analyzed using the classification of errors according to Orton (1983) and Newman (1977). The following are the study's questions: (1) which types of genetic decomposition are employed for investigating how well students comprehend algebraic and trigonometric functions' partial derivatives? (2) How are the results of analyzing students' answers to genetic decomposition? (3) Why do students make errors in determining the partial derivative of algebraic and trigonometric functions?

2. LITERATURE REVIEW

2.1. APOS Theory

The constructivist framework known as APOS theory explains how knowledge of mathematical ideas advances (Martínez-Planell & Trigueros, 2019). This theory is social constructivism that can guide students in building and constructing the material learned (Altieri & Schirmer, 2019; Borji & Voskoglou, 2016; Moon, 2020; Prasetyo et al., 2021). These constructions are the outcome of a mental process that Piaget (1964) first described as reflective abstraction. According to APOS theory, reflective abstraction entails mental structure transforming. After considering a particular problem-solving scenario, someone builds or rebuilds specific mental structures such as actions, processes, objects and schemes. The processes of interiorization, inversion, coordination, encapsulation, and thematization are used to accomplish forming or reconstructing. Each stage of comprehending mathematical concepts is shown by the building of that schema.

Mathematical concepts involve alterations called actions in reaction to stimuli from the environment. An external indication such as a phrase or a method that is learned serves as the basis for characterization. When referring to the idea of a function, an action may add value to an equation and then simplify it to get the outcome. An individual's approach to problem-solving might reveal a framework. One is said to be present at a stage or approaching conception if they are restricted to acting.

Interiorization of the process might result through representations of continued actions. External and internal object changes that enable individuals to consider alterations without really doing them are called actions and processes. As an illustration, the purposeful processes may be considered to be an activity that takes both inputs and outputs. When a person handles challenges might reveal a developed process. By interiorizing actions, reversing already-existing processes and linking two already-existing processes, the process is produced.

The procedure is bundled into an object once the participants can execute alterations. An object's activity can be altered by another function or a pair of functions to produce an alternate function. People's problem-solving techniques can reveal objects. Thus, someone is either at the object stage or has an idea of what an object is. A schema is an arrangement of actions, processes and objects that make sense.

APOS theory is a learning theory that can guide students in building and constructing concepts of the material learned (Borji & Voskoglou, 2016; Moon, 2020). This theory is very useful in understanding learning on various topics. The concept can describe the formation of mathematical knowledge in an individual. The objective to be achieved is to form students' mental construction (Afgani, Suryadi, & Dahlan, 2017; Baye et al., 2021). Dubinsky (1991) described five knowledge constructions from Piaget's reflective abstraction theory, namely:

1. Interiorization is the construction of an internal representation to understand an event.
2. Coordinating any number of processes to create a new process is called coordination.
3. The method by which the object is turned into an object is called encapsulation.
4. The utilization of a previous schema for a new collection of objects by an individual and its growth through the combination are known as generalization.
5. Inversion creates a new process by reversing the original.

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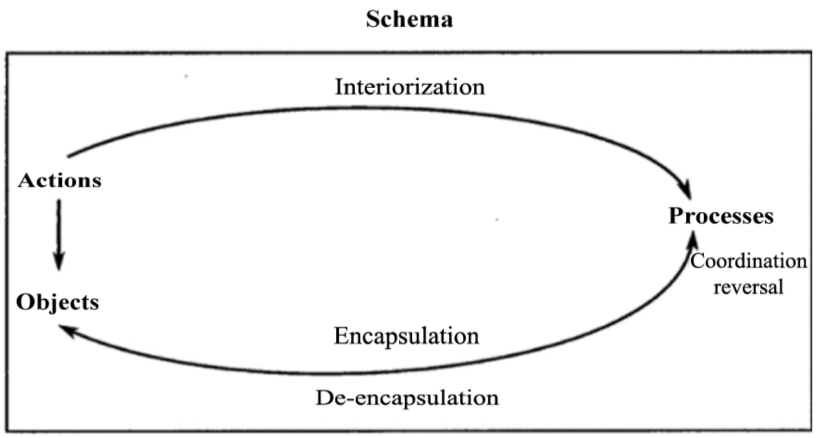

Figure 1. Structure and cognitive mechanisms of APOS theory.

Figure 1 illustrates the formation of a concept in an individual's cognitive structure. Constructing or reconstructing concepts can be achieved through interiorization, reversal, coordination, encapsulation, and thematization mechanisms. The stages of construction activities are actions, processes and objects that form a scheme that indicates the stages of understanding mathematical concepts. Actions are transformations of mathematical objects in response to external cues which can be keywords or memorized processes. Repeated action and reflection on the action can cause interiorization of the process. Processes are internal transformations of objects that allow individuals to think about transformations without actually carrying them out. The results of a process can be seen in the way individuals solve problems. This fact indicates that the individual is at the process stage or has a process conception. Processes are built not only through the interiorization of action but also through the reversal and coordination of two existing processes. Once the process is understood as a whole, the individual can carry out transformations on it, the process is said to have been encapsulated into an object. Subsequently, an individual can de-encapsulate the concept. A coherently organized collection of actions, processes and objects is called a schema.

2.2. Genetic Decomposition

Regarding the principles of mathematics, genetic decomposition is an organized set of mental processes. The mental generation of a mathematical idea is presumably modeled by the parameter (Arnon et al., 2014). The analysis describes mathematical questions according to the APOS theoretical framework (Zwanch, 2019). Formulated as a set of APOS structures and mechanisms, genetic decomposition is an outline of the procedure used to build a specific mathematical idea.

Students are attempting to develop an abstract understanding of mathematics. Materials prepared based on genetic decomposition were proven to improve students' mathematical abstraction abilities in several courses, such as calculus, statistics, linear algebra and real analysis. Some of the genetic decompositions proposed in previous studies include function concepts (Breidenbach, Dubinsky, Hawks, & Nichols, 1992), function composition (Clark et al., 1997; Jojo, 2014) and derivatives (Asiala et al., 1997; Maharaj, 2013). Meanwhile, this study designs a genetic decomposition for the topic of passive derivatives as shown in Figure 2.

Based on Figure 2, students should master the prerequisite material before determining partial derivatives. Understanding the concept of the derivative is the prerequisite material starting with the definition and notation of the function derivative, theorems of a function and applications. Partial derivatives are easily comprehended when students have understood the basic concept due to the use of the rule of one variable function in solving partial derivatives. Some strategies used in solving partial derivatives include restating the power rule to determine partial derivatives, identifying the form of an algebraic or trigonometric function, selecting and using certain rules in determining partial derivatives and linking the concept with limits in determining partial derivatives.

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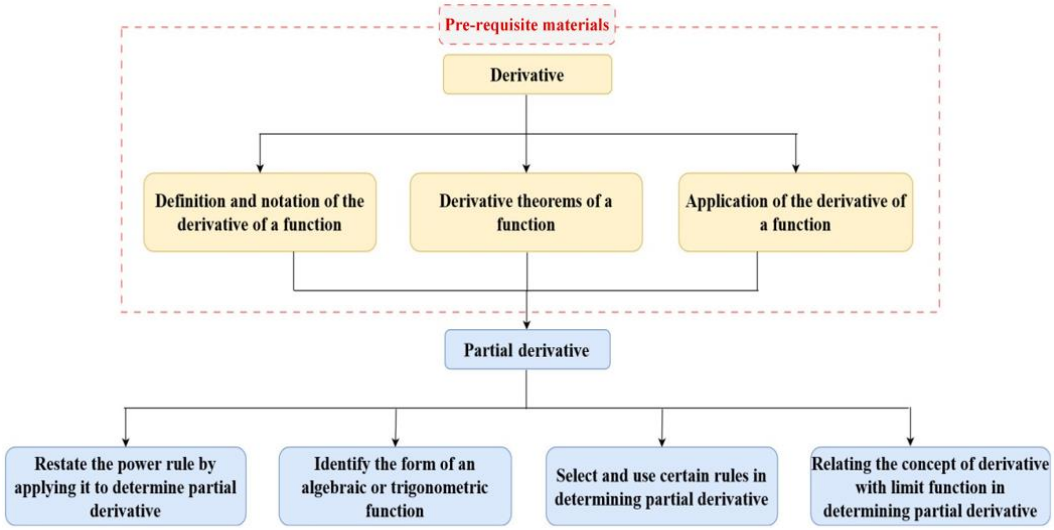

Figure 2. Genetic decomposition in determining the partial derivative.

2.3. Error Classification

This study combines genetic decomposition with error classification in analyzing student answers. Table 1 shows the classification of errors. According to Orton (1983) errors are divided into three categories: (1) arbitrary from failure to pay attention to the limits specified in the question. (2) Structural errors arising from failure to understand the principles underlying the solution. (3) Executive errors due to incorrect manipulation. According to Newman (1977) the types of errors are divided into five, namely: (1) reading errors from failure to understand the meaning of each word, term or symbol in the question. (2) Understanding errors from failure to obtain the needed information in solving the question. (3) Transformation errors resulting from the failure to understand the solution method. (4) Process skill errors due to incorrect use of concepts and performing calculation operations and (5) answer writing errors due to incorrect conclusions.

Table 1. Error classification.

Orton	Newman
Arbitrary errors	Reading errors
Structural errors	Comprehension errors
	Transformation errors
Executive errors	Process skill errors
	Encoding errors

Based on Table 1, reading errors (RE) can be categorized as arbitrary errors (AE), understanding errors (CE) and transformation errors (TE) which are further categorized as structural errors (SE), process skill errors (PE) and answer writing errors (EnE) such as executive errors (ExE). Table 2 outlines the criteria for student answers according to the error classification in Table 1.

Table 2. Classification of students' errors in determining the partial derivative.

Student answer	Errors classification	
	Orton	Newman
The answer shows an ordinary derivative not a partial derivative.	AE	RE
Incorrectly choosing and using the procedures of addition, subtraction, multiplication, division and chain rules in determining partial derivatives.	SE	CE
		TE
Incorrectly performing algebraic calculation operations and writing the conclusion.	ExE	PE
		EnE

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3. METHOD

3.1. Subject

This study comprised third-semester students of the mathematics study program at Universitas Negeri Padang who took the multivariable calculus course. This study involved two classes: in class A, there were 37 students and in class B, there were 36 students. All of these students took the multivariable calculus course in the July–December 2023 semester. The understanding level of students is divided into three categories, namely high, moderate, and low ability as described in Table 3. Each student was chosen at random to represent each of the three skills.

Table 3. Classification of subjects.

High	Medium	Low
Value ≥ Mean + Deviation Standard	Mean – SD ≤ Value < Mean + SD	Value < Mean – Deviation Standard

3.2. Instrument

The investigation of partial derivatives and assessment of students' responses to the following questions were guided by genetic decomposition. Furthermore, an interview guideline was used to determine the thought process for solving the problems. The Pearson Product Moment formula was used so that $r_{xy} = 0.61$ and $r_{xy} = 0.58$ were obtained to determine the validity of the items because $r_{xy} > r_{\text{tabel}}(5\%) = 0.230$, question items number 1a and 1b are said to be valid. Meanwhile, the reliability test used the Cronbach alpha formula obtained $r_{11} = 0.763$ so that it can be concluded that the question includes high-reliability criteria.

1. *Determining* $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$ *from:*
   a. $z = \frac{7x-6y}{5y+6x}$
   b. $z = \sin(4xy + 2x)$

3.3. Data Collection and Analysis

Students worked on two questions consisting of the first partial derivatives. Subsequently, each question's responses were given a rating of 10. The answers provided by the students were classified as correct, partially correct and incorrect. In question number 1, when $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ are determined correctly, the answer is categorized as correct. However, when only $\frac{\partial z}{\partial x}$ or $\frac{\partial z}{\partial y}$ are determined, the answer is classified as partially correct. However, when students only determine the first partial derivative, the answer is categorized as partially correct and others are categorized as wrong. According to Orton (1983) and Newman (1977) the errors made in determining partial derivatives were also analyzed. After analyzing students' answers, interviews were conducted with three students with different abilities. This aims to explore further information related to the answers and obtain the thinking patterns of high-, medium- and low-ability students in determining partial derivatives.

4. RESULTS

The achievement in determining partial derivatives based on APOS theory is shown in Figure 3. In question 1a, 5% of students reached the action stage while 95% reached the action, process, and object stages in determining the first partial derivative. Out of the 95% of students, 41% made mistakes and 54% managed to determine the first partial derivative correctly. In question 1b, 7% reached the action stage while 93% reached the action, process and object stages in determining the first partial derivative. Of the 93% of students, 32% made mistakes and 61% correctly determined the first partial derivative. The data is obtained from students' achievements in each mental construct of APOS theory as shown in Table 4.

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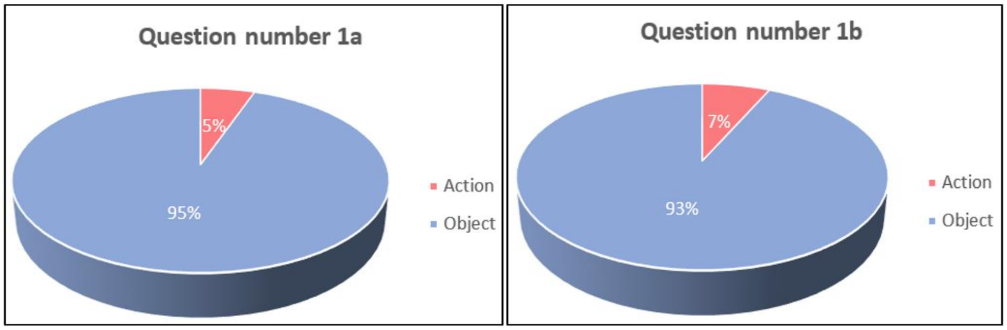

Figure 3. Achievement of APOS stages in question numbers 1a and 1b.

Table 4. Indicators of student concept understanding based on APOS theory.

Topic	APOS stages	Indicator
Partial derivatives	Action	Students can 1) Identify what is known and asked about in the problem. 2) Restate the power rule by applying the rule to determining partial derivatives of algebraic or trigonometric functions.
	Process	Students can 1) Presenting an algebraic function into a certain form so that the addition or substraction rules of several terms can be used. 2) Selecting the procedure of addition or subtraction rules in determining partial derivatives of algebraic or trigonometric functions. 3) Using the addition or subtraction rule procedure in determining the partial derivative of algebraic or trigonometric functions combined with using the power rule repeatedly on each term appropriately according to the procedure.
	Object	Students can 1) Identify the form of an algebraic or trigonometric function, either composition, product, and quotient of two functions or chain rule according to certain properties appropriately. 2) Select and use certain rules appropriately according to the results of function form classification.
	Schema	Students can 1) Relate the concept of derivative to the function limit in determining the partial derivative of algebraic or trigonometric functions. 2) Select and use certain rules on the concept of limit function appropriately according to the procedure.

In addition to student achievement data, errors in determining partial derivatives were also analyzed as shown in Table 5. In solving question 1a, AE or RE were not made since students comprehended the meaning of each word and paid attention to the limits. Furthermore, 17.81% of students made SE consisting of 5.48% CE and 12.33% TE. This is because students failed to understand the principles fundamental to the solution such as incorrectly using the procedures of addition, subtraction, multiplication, division and chain rules in determining the first partial derivative. Finally, 28.77% of students made ExE comprising 15.07% PE and 13.70% EnE. These errors arose from incorrect manipulations such as incorrectly performing algebraic operations and drawing conclusions.

Table 5. Percentage of student errors in determining the partial derivative.

Questions	Errors classification							
	Orton			Newman				
	AE	SE	ExE	RE	CE	TE	PE	EnE
1a	-	17.81	28.77	-	5.48	12.33	15.07	13.70
1b	1.37	10.96	27.40	1.37	8.22	2.74	27.40	-

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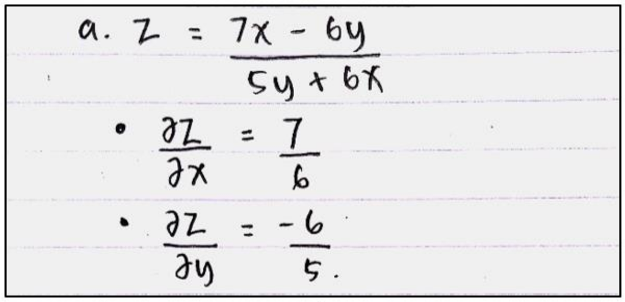
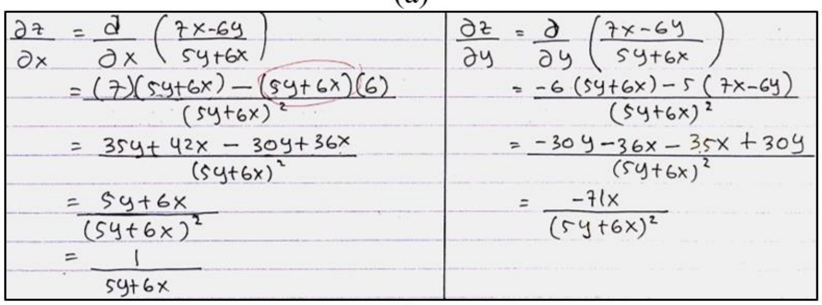
In question number 1b, 1.37% of students made AE since the question instructions were not understood. About 10.96% made SE consisting of 8.22% CE and 2.74% TE since students failed to obtain information in determining the first partial derivative. Moreover, 27.40% made ExE and PE due to the failure to perform algebraic calculation operations.



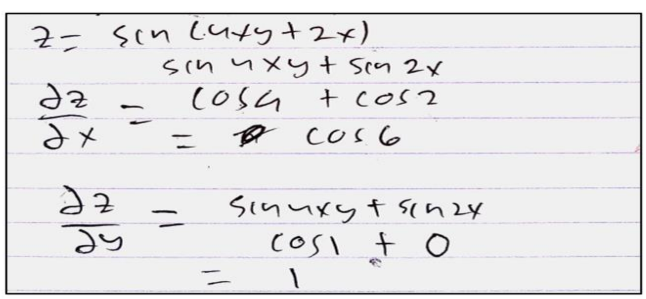
(a)



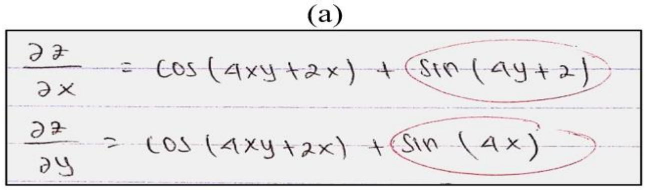
(b)

Figure 4. Student errors on question number 1a.

In Figure 4a, students determine $\frac{\partial z}{\partial x}$ by directly reducing the variables in the numerator and denominator with $\frac{\partial z}{\partial y}$. As a result, the level of error was classified as CE and the idea of the partial derivative was not recognized. Figure 4b shows that the student used the division rule $\left(\frac{u}{v}\right)$ in solving question number 1a. However, a mistake is made in determining the derivative $7x - 6y$ and $5y + 6x$ on x. In this case, the student only eliminated the variable x by writing y against x. The same method is also used when the derivative of $7x - 6y$ and $5y + 6x$ on y is determined. According to Orton (1983) students make TE and these two types of errors include SE.



(a)



(b)

Figure 5. Student's errors on question number 1b.

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Based on Figure 5a, students rewrite the given function into $\sin(4xy) + \sin(2x)$ and the derivative of each is determined. As a result, the level of error was classified as CE and the idea of the partial derivative was not recognized. In contrast to Figure 5b, students successfully determined the derivative of $\sin(4xy + 2x)$ but made an error when determining the derivative in parentheses. The following illustrates that the students were unable to properly adapt the chain rule technique to trigonometric functions.

4.1. Answers of High Ability Student (HAS)

Figure 6 shows that the High Ability Student (HAS) chose and used the division rule in solving question number 1a. MT determines $\frac{\partial z}{\partial x}$ before continuing with $\frac{\partial z}{\partial y}$. An interview was conducted to explore HAS's understanding of solving question number 1a as follows: In question 1a, we know the z-function which consists of two variables. We are asked to determine the first partial derivative. So, to solve it, we first determine $\frac{\partial z}{\partial x}$. It is assumed that the variable y is constant while finding the derivative of z with the value of x. In this case, the z-function is division, so we can say $\frac{u}{v}$.

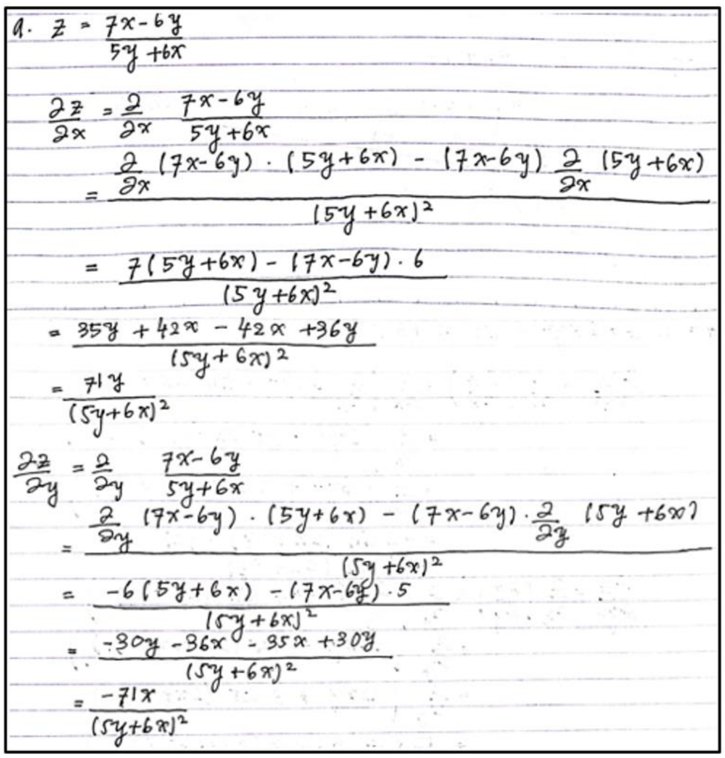

Figure 6. HAS's answer to question number 1a.

Based on the interview, HAS selects and uses the addition and subtraction rule procedures in determining partial derivatives of algebraic functions identifying the form of an algebraic function as well as selecting certain procedures according to the classification of the function form. As a result, to find the first partial derivative of an algebraic function, the action, process, and object phases are established.

4.2. Answers of Medium Ability Students (MAS)

In Figure 7, Medium Ability Students (MAS) generalizes $u = 7x - 6y$ and $v = 5y + 6x$ before using the division rule $\left(\frac{u}{v}\right)$ in determining the first partial derivative. However, the answer is not correct because v^2 is wrong and $v = 5y + 6x$ is formalized. An interview is carried out to further explore MAS's understanding of solving question 1a. First, suppose $u = 7x - 6y$ and $v = 5y + 6x$. Then, we find $\frac{\partial z}{\partial x}$. In this case, I determined the derivative using

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the $\frac{u}{v}$ rule. The derivative of $u = 7x - 6y$ concerning x is 7 and the derivative of $v = 5y + 6x$ concerning x is 6. Next, we substitute the values into $\frac{u'v-uv'}{v^2}$ to get $\frac{71y}{(5y-6x)^2}$. If $\frac{\partial z}{\partial y}$ is derived concerning y. The derivative of $u = 7x - 6y$ with respect to y is –6 and the derivative of $v = 5y + 6x$ concerning y is 5. So, the result is $-\frac{71x}{(5y-6x)^2}$.

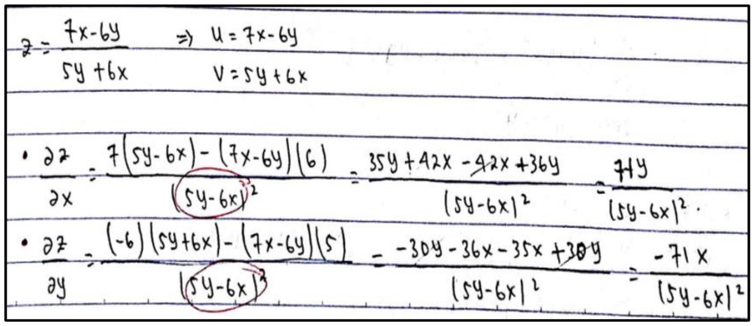

Figure 7. MAS's answer to question number 1a.

Based on the interview, MAS can identify and ask questions, select the addition and subtraction rule procedures for determining partial derivatives of algebraic functions, identify the form of an algebraic function and select certain procedures according to the classification results. EnE and ExE error types are made in writing v^2 since the division rule procedure is incorrect. After being confirmed, MAS realized a mistake in writing v^2 which should be $(5y + 6x)^2$. Hence, while figuring out the first partial derivative of an algebraic function, MAS is aware of the action, process and object phases.

4.3. Answers of Low Ability Students (LAS)

Low Ability Students (LAS) determine the first partial derivative using the concept of limit as shown in Figure 8. However, a mistake was made in writing $\sin\left(\frac{7x-6y}{5y+6x}\right)$ since $\frac{\partial z}{\partial x}$ became wrong and $\frac{\partial z}{\partial y}$ was not determined. An interview was carried out to further explore LAS's understanding of solving question number 1a as follows. In this problem, we are given the function $z = \frac{7x-6y}{5y+6x}$. We are asked to determine its partial derivative. LAS solved it using the limit of the z-function. In this case, $\frac{\partial z}{\partial x} = \lim_{x\to 0} \sin\left(\frac{7x-6y}{5y+6x}\right)$. Because LAS does not understand the partial derivative solution they used the limit.

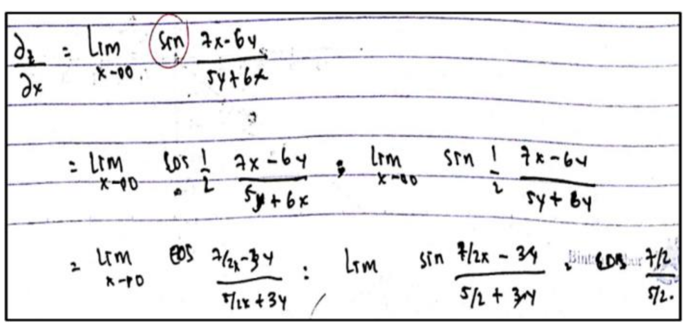

Figure 8. LAS's answer to question number 1a.

Based on the interview, LAS can choose certain procedures for determining partial derivatives of algebraic functions. However, the partial derivative procedure has not been performed using the concept of the limit function correctly due to PE and ExE errors. The action and process stages are comprehended in determining the first partial derivative of an algebraic function. The following summarizes HAS, MAS and LAS's achievements in solving question 1a.

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Table 6. Achievements of MT, MS, and MR in solving question number 1a.

Subjects	APOS stages and indicators				
	Action	Process		Object	
	1	2	3	1	2
HAS	√	√	√	√	√
MAS	√	√	√	√	-
LAS	√	√	-	-	-

In question 1a, there are two indications at the process and object phases and one indicator at the action phase. Table 6 shows that HAS achieved all indicators at the action, process and object stages. During the action and process phases, MAS fulfilled its targets during this time. At the object stage, MAS only fulfills 1 indicator while LAS fulfills the indicators at the action and process stages.

5. DISCUSSION

This study presents an examination of students' thinking processes within the APOS theory framework when determining partial derivatives of algebraic and trigonometric functions. The theory analyzes the thinking ability of an individual in solving mathematical questions (Widada, Herawaty, Nugroho, & Anggoro, 2019). The APOS theory framework describes concept learning, curriculum design and evaluation (Karama, 2021). When figuring out the partial derivatives, HAS, MAS and LAS maintain knowledge of the action, process and object phases. However, MAS understanding at the object stage is still lacking as well as LAS understanding at the process and object stages. The following demonstrates that low ability students require extra assistance in comprehending mathematical concepts. Teachers must provide them with materials based on what they are seeking to acquire knowledge (Bayu, Fauzan, & Armiati, 2023).

Math problems are attempted differently by each student (Harisman, Dwina, Nasution, Amiruddin, & Syaputra, 2023). Students must use their capacity for thought when solving mathematical problems to ensure that no mistakes are made (Azizah, Fauzan, & Harisman, 2022; Kögce, 2022). Nevertheless, we discovered several mistakes were made in determining partial derivatives such as incorrectly selecting and using the procedures of addition, subtraction, multiplication, division and chain rules. Siyepu (2015) also found several types of errors in determining the derivative of trigonometric functions including conceptual errors showing failure to understand a concept, errors in procedure that arise from the student's inability to execute algorithms or activities, students making interpretive errors when they acquire an idea incorrectly and errors in linear extrapolation that take place when students expand this property. This only holds in the case of linear functions. The error categorization we employed in this instance differed from Mukhni and Utami's (2023) categorization of students' errors when solving vectors using the Castellan stage. The result revealed that most of the students were not careful when operating vectors. Not only that, the students were also mistaken in determining the angle between two vectors and projecting vectors orthogonally. This represents a type of conceptual error. The reason behind this is that when handling vector issues students are unable to select and apply a suitable formula. Karama (2021) found that students do not understand the processes to determine the first derivative using the concept of limit. A few people are not conscious of the connection at a point, the tangent line $x = a$, or the visual representation of the derivative because students do not understand the prerequisite material such as function, limit and gradient (Borji et al., 2018). Furthermore, it might be challenging to determine which function allows the chain rule to be applied and to understand a function's derivative (Jojo, Maharaj, & Brijlall, 2012).

Using APOS theory, Burns-Childers and Vidakovic (2018) looked at how to comprehend the connection between a quadratic function's edge and derivative. The findings show that students' understanding at the schema stage is lacking. Numerical methods are also used to identify crucial values and suitable regions for the cusp. Widada et al. (2019) adopted genetic decomposition in APOS theory to explore the derivative of a function. The outcomes demonstrate that students can condense the process of the provided function's attributes periodically in the domain$h$. As a result, an object is created about the function graph h sketching and this feature demonstrates that students are at the intermediate level. Genetic decomposition provides a framework used to analyze student understanding of mathematics learning.

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Martínez-Planell and Trigueros (2019) implemented APOS-based two-variable function learning through three cycles. Through the use of APOS-based exercises, students were capable of to visualize curves, surfaces and other mathematical concepts. Dynamic images of two-variable functions were generated without performing explicit calculations. Significant linkages were also made with other mathematical knowledge to rationalize as well as organize various activities without depending on methods or facts that were committed to memory. Thus, it is possible to enhance students' mathematics comprehension by using various APOS analysis cycles.

The student's awareness of employing the chain method to find the derivative of trigonometric functions was examined by Jojo et al. (2012). The key to understanding the chain rule is function composition. Offering instructional approaches for the topic is informed by the APOS theory framework which also supports the chain rule. Maharaj (2013) reported that most students struggled to apply the derivative rule. This phenomenon arises from students lacking the requisite cognitive structures at the stages of process, object and schema. According to Moru (2020), the majority of mental constructs developed during the learning process of derivatives primarily include actions. The algebraic form makes the idea of the derivative as a limit clear.

APOS theory can also be used to explore the understanding of two variable functions (Martínez-Planell, Gaisman, & McGee, 2015). This demonstrates how crucial mental construction is to recognizing the idea of derivatives. According to Afgani et al. (2017), the mental construct that is produced during problem solving in mathematics might indicate whether a given issue is successfully solved or not. The application starts with designing genetic decomposition based on understanding related to building mathematical concepts (Borji & Martínez-Planell, 2023). Each student has different characteristics for understanding mathematical concepts and the mental construction carried out has different stages (Tatira, 2021). Students' behavior will alter as a result of this learning process (Musdi, Syaputra, & Harisman, 2024). Learning can take many different forms (Harisman, Dwina, & Tasman, 2022) including process to action, object to process and process to object (Kurniawan, Sutawidjaja, As'ari, Muksar, & Setiawan, 2018). Consequently, to get students interested in learning mathematics, teachers need to select and implement the right teaching methods and learning resources (Harisman, Mayani, Armiati, Syaputra, & Amiruddin, 2023).

6. CONCLUSION AND RECOMMENDATIONS

The conceptualization of the action, process and object phases in deriving the partial derivatives was expressed by students with high, medium and low abilities. However, the understanding of medium ability students at the object stage was still lacking as well as low ability students at the process and object stages. Several errors were made in determining partial derivatives, including incorrectly selecting and using the procedures of addition, subtraction, multiplication, division and chain rules. The dominant types of errors in determining partial derivatives were ExE and PE. Future studies were expected to modify the genetic decomposition of partial derivatives and expand the classification of errors in data analysis. A connection between erroneous categorization and the APOS cognitive framework was reinforced by this genetic decomposition enhancement which also offered further understanding and proof of the mental constructs. Moreover, the addition of more detailed error types explained the development of mental constructs.

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INSTITUTIONAL REVIEW BOARD STATEMENT

The Ethical Committee of the Padang State University, Indonesia has granted approval for this study on 3 July 2023 (Ref. No. 9501/UN35.1/KP/2023).

TRANSPARENCY

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

COMPETING INTERESTS

The authors declare that they have no competing interests.

AUTHORS' CONTRIBUTIONS

Conceptualization, Y.Y.; methodology and formal analysis, Y.Y., YL.S., and A.A.; writing original draft preparation, Y.Y., YL.S., and K.A.H.; writing, review and editing, Y.Y., YL.S., A.A and S.S. All authors have read and agreed to the published version of the manuscript.

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